MathLinks

FRACTIONS 1
STUDENT PACKET

FRACTION CONCEPTS

Parent (or Guardian) signature ___________________________________

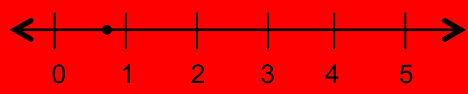
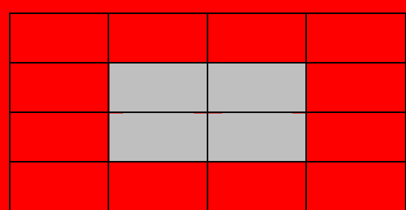
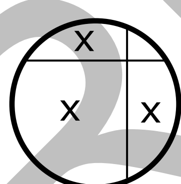
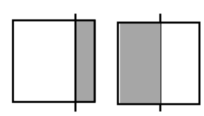
MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. (See section 1.5.) Key mathematical vocabulary is underlined throughout the packet.

area model for fractions	benchmark fraction
denominator	equivalent fractions
linear model for fractions	multiplication property of 1
numerator	unit fraction

OPENING PROBLEM: THREE-FOURTHS

Which of these could be a representation of $\frac{3}{4}$? Explain using complete sentences.

1. 	2. There are 18 girls in a class of 24.
3. 	4. Four friends want to share 3 cans of fruit juice equally. How much will each friend get?
5. 	6. 

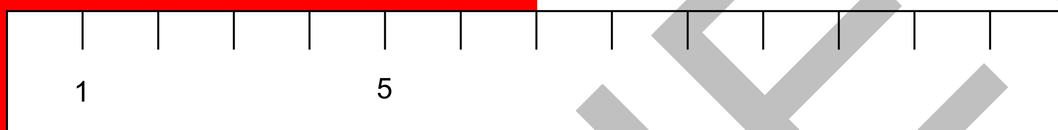
FRACTION STRIPS

We will use a linear model to explore fraction equivalence. We will use sense-making strategies to order fractions.

GETTING STARTED

1. Label centimeters on the ruler below.

   What goes on the left edge? What goes on the right edge?



Use the ruler above to answer the following questions.

2. What is the sum of 5 centimeters and 3 centimeters?

3. What is the difference between 9 centimeters and 4 centimeters?

4. How long are 3 groups of 4 centimeters?

5. How many groups of 2 centimeters are in 8 centimeters?

6. What fractional part of 10 centimeters is 2 centimeters?

7. In this lesson, you will see the "big 1" used as a reminder of fractions that are equal to 1.

   One example is $\frac{4}{4}$. Write three more fractions with a value of 1.

8. What is the result when a number is multiplied by 1?

   Give an example.

9. What is the result when a number is divided by 1?

   Give an example.

FRACTION STRIPS 1

Follow your teacher's directions.

PRACTICE 1

1. What is the "big 1"?

2. Write inequalities to compare the unit fractions $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$. Explain your reasoning.

$$\frac{\quad}{\quad} < \frac{\quad}{\quad} < \frac{\quad}{\quad}$$

3. Write fractions with denominators of 2, 4, and 8 that are equivalent to 1.

$$1 = \frac{\quad}{2} = \frac{\quad}{4} = \frac{\quad}{8}$$

4. Use the "big 1" to write fractions that are equivalent to $\frac{1}{2}$.

$$\frac{1}{2} \bullet \frac{\quad}{\quad} = \frac{\quad}{4} \quad \text{and} \quad \frac{1}{2} \bullet \frac{\quad}{4} = \frac{\quad}{8}$$

5. Use the "big 1" to write a fraction with a denominator of 8 that is equivalent to $\frac{3}{4}$.

$$\frac{3}{4} \bullet \frac{\quad}{\quad} = \frac{\quad}{8}$$

6. Use the "big 1" to write a fraction with a denominator of 2 that is equivalent to $\frac{2}{4}$.

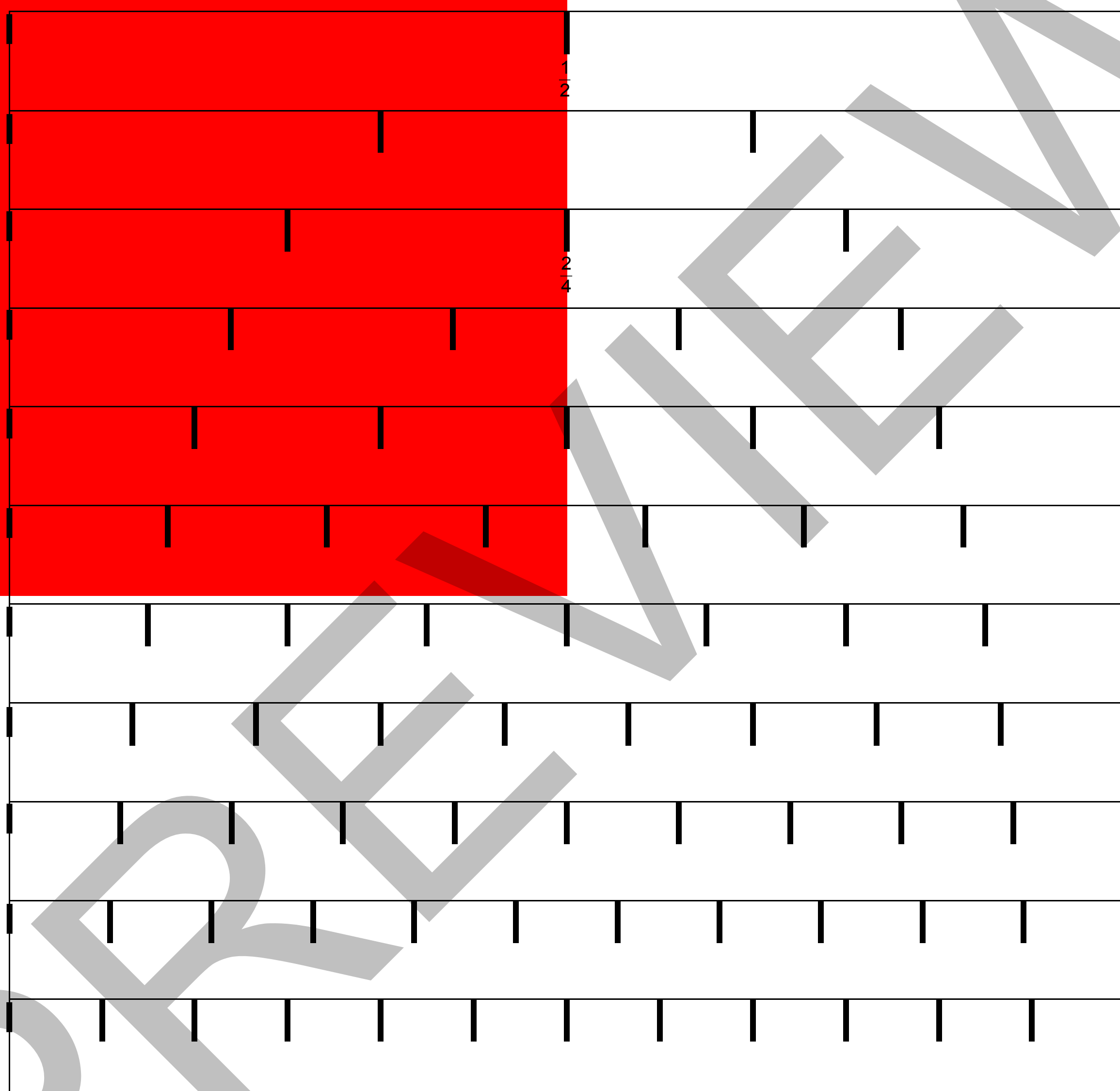
$$\frac{2}{4} \div \frac{\quad}{\quad} = \frac{\quad}{2}$$

7. How are fourths related to halves?

8. How are fourths related to eighths?

FRACTION STRIPS 2

Follow your teacher's directions.



PRACTICE 2

1. Write three fractions that are equivalent to 0. What is the same about each of these fractions?

2. Write three fractions that are equivalent to 1. What is the same about each of these fractions?

3. On your array on the previous page, mark the points that represent $\frac{1}{3}$, $\frac{2}{6}$, $\frac{3}{9}$, and $\frac{4}{12}$. What do you notice about these locations, and what does it tell you about the values of these fractions?

   In general, how can you tell from the fraction array if fractions are equivalent?

4. Write three fractions that are equivalent to $\frac{1}{2}$. What is the relationship between the numerator and denominator in each of these fractions?

5. Write inequalities to compare these fractions: $\frac{1}{2}$, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{3}$.

   $\boxed{\frac{\quad}{\quad}} < \boxed{\frac{\quad}{\quad}} < \boxed{\frac{\quad}{\quad}} < \boxed{\frac{\quad}{\quad}} < \boxed{\frac{\quad}{\quad}}$

   In general, if two fractions both have a numerator equal to 1 (unit fractions), how can you tell which fraction has the greater value?

6. Write inequalities to compare these fractions: $\frac{4}{6}$, $\frac{5}{6}$, $\frac{1}{6}$, and $\frac{3}{6}$.

   $\boxed{\frac{\quad}{\quad}} < \boxed{\frac{\quad}{\quad}} < \boxed{\frac{\quad}{\quad}} < \boxed{\frac{\quad}{\quad}}$

   In general, if two fractions have the same denominator, how can you tell which fraction has the greater value?

STRATEGIES FOR ORDERING FRACTIONS

Follow your teacher's directions. All fractions are based the same size whole.

Word List			
numerator	denominator	benchmark	unit

	Fractions	Ordering Strategy
(1)		These are called __________ fractions. Explain:
(2)		These fractions all have a common __________. Explain:
(3)		These fractions all have a common __________. Explain:
(4)		Simplify $\frac{3}{6} \rightarrow \boxed{\frac{}{}}$. This is called a __________ fraction because it is easily recognizable. Explain:

5. Use section 1.5 to help you write explanations and examples for <u>numerator</u>, <u>denominator</u>, <u>benchmark fraction</u>, and <u>unit fraction</u> in My Word Bank.

PRACTICE 3

1. Estimate the location of each number on the number line: 0 1 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{6}{8}$

What benchmark fractions did you locate on your number line first?

Explain how you located $\frac{6}{8}$ on the number line.

2. Estimate the location of each number on the number line: 0 1 $\frac{3}{4}$ $\frac{3}{9}$ $\frac{3}{7}$

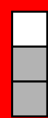
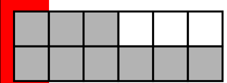
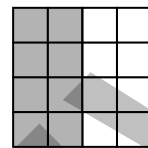
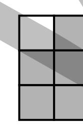
Explain how you located $\frac{3}{7}$ and $\frac{3}{9}$ on the number line.

3. Estimate the location of each number on the number line: 0 1 $\frac{3}{6}$ $\frac{1}{6}$ $\frac{4}{6}$

Explain how you located $\frac{1}{6}$ and $\frac{4}{6}$ on the number line.

THE FLOWER GARDEN PROBLEM

1. Four students have gardens of different sizes. Below are scale drawings of the gardens, where each square represents one square yard. The shaded portions below represent the part of each garden that is planted. Complete the table.

Student	Colin	Indy	Sam	Blue
The garden				
Number of square yards planted				
Number of square yards in the garden				
Fraction of the garden that is planted				

2. Indy says that his garden has the largest fractional part planted. Colin, Sam, and Blue disagree with Indy. Settle the disagreement. Use different strategies (pictures, “big 1” calculations, or ordering strategies) to support your answer.

Who has the greater fractional part planted: Colin or Indy?	Who has the greater fractional part planted: Sam or Indy?	Who has the greater fractional part planted: Blue or Indy?

3. Whose garden has the greatest fractional part planted? _____ How do you know?

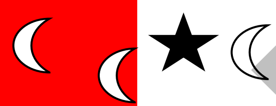
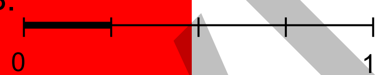
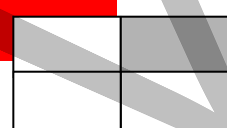
4. Indy says that since he has 9 squares planted, he has the most out of everyone. Blue disagrees. Write an explanation justifying each person’s statement.

RENAMING FRACTIONS

We will represent fractions greater than one as mixed numbers and as improper fractions.

GETTING STARTED

Match each model for illustrating fractions to one picture and one verbal description. Refer to section 1.5 for help with the vocabulary as needed.

Models		Pictures	Verbal Descriptions
_____ _____ 1.	Linear model	A. 	X. $\frac{1}{4}$ of the big rectangle is shaded.
_____ _____ 2.	Area Model	B. 	Y. $\frac{1}{4}$ of the shapes are stars.
_____ _____ 3.	Set model	C. 	Z. $\frac{1}{4}$ of the length is bold.

Interpret the meaning of the numerator and denominator for each model.

	Model	The numerator is shown as:	The denominator is shown as:
4.	Linear model		
5.	Area model		
6.	Set model		

7. Write an explanation and example of linear model for fractions and area model for fractions in My Word Bank. Use section 1.5 if needed.

BROWNIE PROBLEMS

Follow your teacher's directions to solve these "fair share" problems.

(1) ____ friends want to share ____ brownies so that each one gets the same amount. How much can each friend have?	(2) ____ friends want to share ____ brownies so that each one gets the same amount. How much can each friend have?
(3) ____ friends want to share ____ brownies so that each one gets the same amount. How much can each friend have?	(4) ____ friends want to share ____ brownies so that each one gets the same amount. How much can each friend have?

PRACTICE 4

Refer to section 1.5 for the definitions of proper fraction, improper fraction, and mixed number.

Circle the word that correctly identifies each number below.

1. $\frac{3}{8}$	proper fraction improper fraction mixed number	2. $\frac{11}{2}$	proper fraction improper fraction mixed number	3. $2\frac{5}{11}$	proper fraction improper fraction mixed number
4. $1\frac{3}{8}$	proper fraction improper fraction mixed number	5. $\frac{11}{11}$	proper fraction improper fraction mixed number	6. $\frac{27}{11}$	proper fraction improper fraction mixed number

Complete the table below. Each rectangle represents one whole cracker.

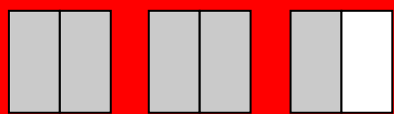
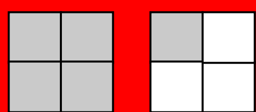
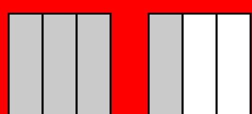
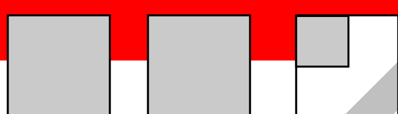
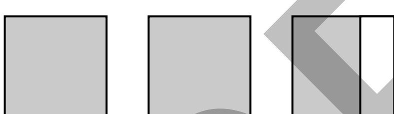
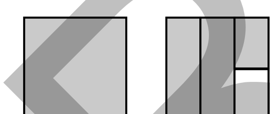
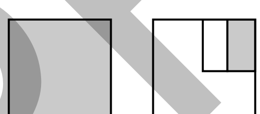
Amount in words	**Shade the appropriate amount (there may be extra squares)**	**Write the number**
7. One-half of a cracker	☐ ☐ ☐ ☐	
8. One and one-half crackers	☐ ☐ ☐ ☐	
9. Two and one-half crackers	☐ ☐ ☐ ☐	
10. Three-halves crackers	☐ ☐ ☐ ☐	

11. Which word descriptions above represent the same amount of crackers?

12. Refer to the picture you made for problem 9 (two and one-half crackers). How many halves is this?

PRACTICE 5

Represent each picture below with numerical expressions. Words are included in the example for interpretation, but you do not need to write each expression in words. Each rectangle represents one whole cracker.

Shaded Crackers	Sum	Mixed Number	Conversion	Improper Fraction
Example 	$2 + \frac{1}{2}$ two plus one-half	$2\frac{1}{2}$ two and one-half	$\frac{2}{2} + \frac{2}{2} + \frac{1}{2}$ two halves plus two halves plus one-half	$\frac{5}{2}$ five halves
1. 				
2. 				
3. 				
4. 				
5. 				
6. 				

7. Molly thinks that the mixed number represented in problem 5 above is $1\frac{3}{4}$ because one whole is shaded, and 3 out of 4 parts of a whole are shaded. Critique Molly's reasoning.

RENAMING SHORTCUTS

Follow your teacher's directions for problems 1-6.

(1)	(2)
(3)	
(4)	(5)
(6)	

Change each mixed number into an improper fraction.

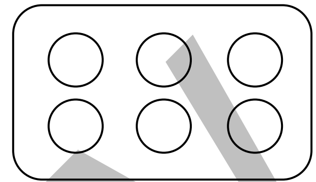
7. $4\frac{3}{5}$	8. $2\frac{1}{6}$	9. $8\frac{3}{7}$

Change each improper fraction into a mixed number.

10. $\frac{8}{3}$	11. $\frac{23}{4}$	12. $\frac{42}{9}$

PRACTICE 6

This diagram represents one whole pack of muffins or cupcakes.



1. Shade $\frac{1}{2}$ of the pack.

2. Draw sketches to represent the following:

Number of packs of muffins	$\frac{3}{2}$ packs	$1\frac{1}{6}$ packs	$1\frac{1}{3}$ packs
Sketch	a.	b.	c.

3. If 3 cupcakes represent three-fourths of a pack of muffins, draw sketches to represent:

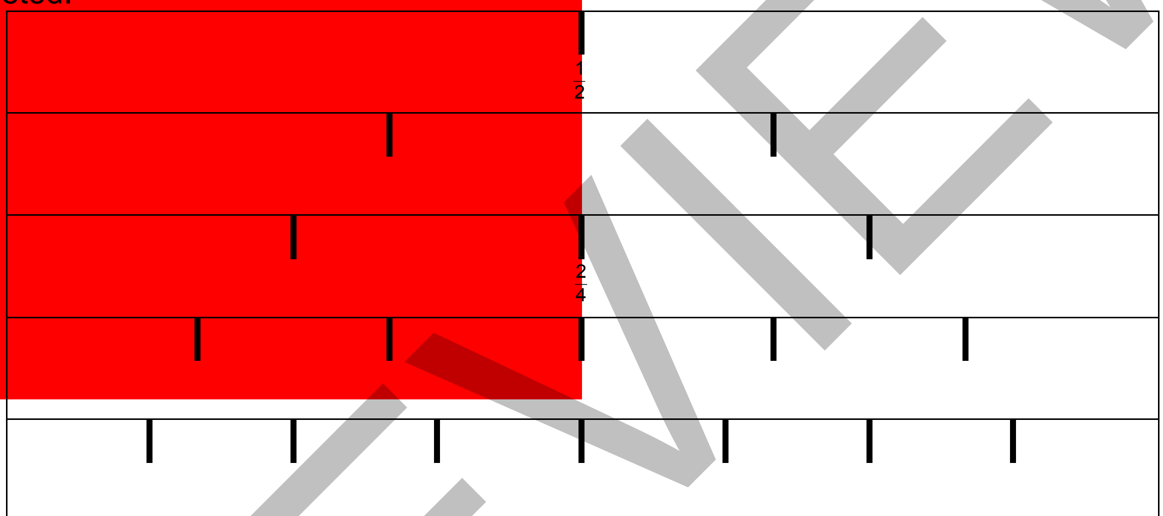
Number of packs of cupcakes	1 whole pack	$1\frac{1}{2}$ packs	$\frac{9}{8}$ packs
Sketch	a.	b.	c.

EQUIVALENT FRACTIONS

We will use diagrams to illustrate equivalent fractions. We will connect the diagrams to computations. We will compare fractions in a problem solving setting.

GETTING STARTED

1. Write in fractions to complete this portion of a fraction array. Careful! Some rows have been deleted.



2. Use the array in problem 1 to name two different pairs of equivalent fractions. How do you know each pair is equivalent?

3. How would you interpret the fraction array as a linear model?

4. How would you interpret the fraction array as an area model?

EQUIVALENT FRACTIONS WITH AREA MODELS

Ms. Jetter asked her students to draw diagrams to show that $\frac{2}{3}=\frac{4}{6}$. Follow your teacher's directions to complete this page.

(1)

(2)

(3)

(4)

(5)

(6)

(7)

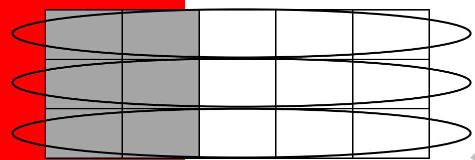
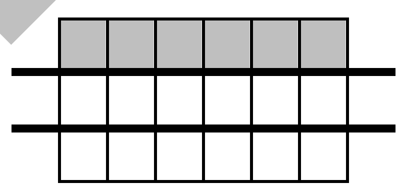
(8)

(9)

(10)

PRACTICE 7

Use area diagrams and the multiplication property of 1 to write equivalent fractions. Include these words in My Word Bank. Use section 1.5 for help if needed.

	Equivalent fractions	Diagram	"Big 1" calculation
1.	$\frac{2}{5} = \frac{6}{15}$		$\frac{2}{5} \bullet \frac{\ }{\ } = \frac{6}{15}$
2.		Lines show groupings. 	$\frac{12}{20} \div \frac{4}{4} = \boxed{\frac{\ }{\ }}$
3.	$\frac{2}{4} = \frac{1}{2}$		
4.			$\frac{2}{3} \bullet \frac{3}{3} = \boxed{\frac{\ }{\ }}$
5.			
6.	$\frac{3}{4} = \frac{6}{8}$		

PRACTICE 7
(Continued)

7. Diagrams 1 and 2 illustrate that $\frac{1}{3} = \frac{3}{9}$.

Diagram 1

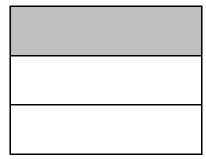
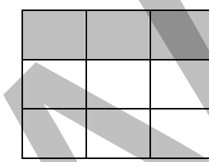


Diagram 2



How are they the same?

How are they different?

Write a statement of equivalence using multiplication and the "big 1."

8. Diagrams 3 and 4 illustrate that $\frac{8}{20} = \frac{2}{5}$.

Diagram 3

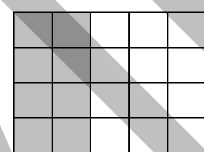
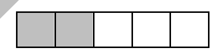


Diagram 4

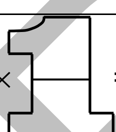
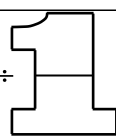


How are they the same?

How are they different?

Write a division equivalence statement using the "big 1."

Multiply or divide by a form of the "big 1" to complete each equivalence statement. Draw diagrams if needed.

9. $\frac{1}{2} \times$  $= \frac{}{6}$	10. $\frac{10}{25} \div$  $= \frac{2}{5}$	11. $\frac{9}{15} \div \qquad = \frac{3}{}$
12. $\frac{3}{} \times \qquad = \frac{9}{12}$	13. $\frac{2}{9} \times \qquad = \frac{}{45}$	14. $\frac{}{56} \div \qquad = \frac{2}{7}$

WRITING FRACTIONS IN "SIMPLEST FORM"

Refer to section 1.5 as needed for vocabulary and procedures.

These ARE <u>factors</u> of 24.	These ARE NOT <u>factors</u> of 24.
1, 2, 3, 4, 6, 8, 12, 24	5, 14, 25, 48, 72

1. A <u>factor of a number</u> is ____ .

Write all the factors of the given numbers.

2. 12	3. 15	4. 6	5. 30

These ARE fractions in "simplest form."	These ARE NOT fractions in "simplest form."
$\frac{1}{2}, \frac{2}{3}, \frac{14}{17}, \frac{5}{3}, \frac{4}{25}$	$\frac{2}{4}, \frac{10}{15}, \frac{6}{16}, \frac{20}{6}, \frac{4}{400}$

6. We say that a fraction is in "simplest form" when ______________________________

____.

Write each fraction in simplest form. Show a "big 1" calculation.

7. $\frac{6}{8}$	8. $\frac{6}{12}$	9. $\frac{12}{15}$
10. $\frac{10}{25}$	11. $\frac{9}{36}$	12. $\frac{20}{36}$
13. $\frac{20}{70}$	14. $\frac{12}{20}$	15. $\frac{48}{100}$

16. Annika thinks $\frac{6}{24} = \frac{3}{12}$. Is she correct? _____ Is this simplest form? _____ Explain.

COMPARING FRACTIONS USING COMMON DENOMINATORS

Refer to section 1.5 as needed for vocabulary and procedures.

These ARE multiples of 6.	These ARE NOT multiples of 6.
6, 12, 18, 24, 30, 60	1, 2, 3, 7, 14

1. A multiple of a number is ______________________________

______________________________ .

Write the first six multiples of each number.

2. 2	3. 4	4. 6	5. 10

These fraction pairs HAVE a common denominator.	These fraction pairs DO NOT HAVE a common denominator.
$\frac{8}{12}$ and $\frac{9}{12}$ $\frac{3}{30}$ and $\frac{18}{30}$ $\frac{5}{20}$ and $\frac{22}{20}$	$\frac{3}{5}$ and $\frac{3}{10}$ $\frac{4}{40}$ and $\frac{2}{20}$ $\frac{5}{6}$ and $\frac{6}{5}$

6. Two fractions have a common denominator when ______________________ .

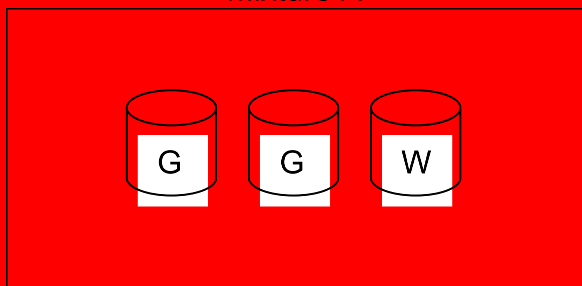
For problems 7-12, rewrite both fractions with a common denominator. Circle the fraction with the greater value.

7. $\frac{1}{2}$ and $\frac{3}{4}$	8. $\frac{2}{3}$ and $\frac{3}{4}$	9. $\frac{7}{8}$ and $\frac{5}{6}$
10. $\frac{1}{4}$ and $\frac{3}{5}$	11. $\frac{5}{3}$ and $\frac{7}{5}$	12. $\frac{3}{8}$ and $\frac{5}{12}$

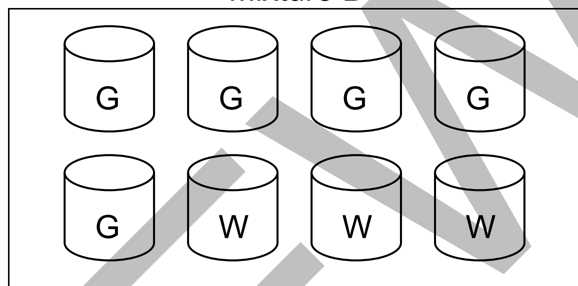
JUICE MIXTURES

A grape juice recipe is made with parts of concentrated grape juice (G) and parts of water (W). Here are four pictures that show recipes for making grape juice. Which is the most “grapey?” Show your reasoning using TWO different strategies.

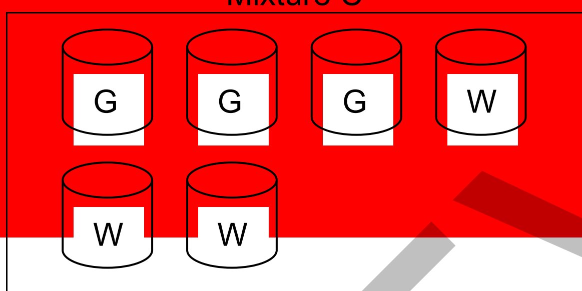
Mixture A



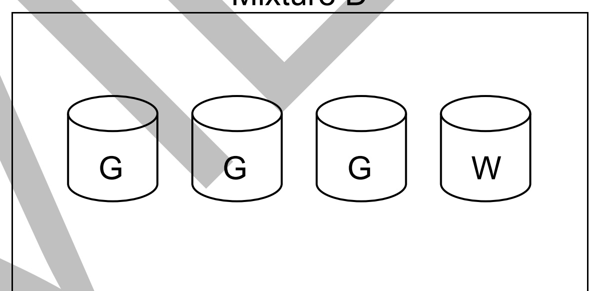
Mixture B



Mixture C



Mixture D



REVIEW

GAME: FRACTION RUMMY

This game is for 2-4 players. Each group will need 40-48 cards. For step 1, each group creates its own set of Rummy Cards (cut up R2). To bypass step 1, use Fraction Cards 1-3 (cut up R3abc).

1. Groups create 10-12 sets of 4 equivalent fractions. Examples of two sets are:

$\frac{1}{2}$	$\frac{4}{8}$	$\frac{3}{6}$	$\frac{5}{10}$

$\frac{3}{4}$	$\frac{75}{100}$	$\frac{6}{8}$	$\frac{18}{24}$

2. Record two of the sets of equivalent fraction cards that you made (or the set you are given).

3. Establish the game rules and play. One variation is:

- The dealer shuffles the deck and deals seven cards to each player. The next card is turned face-up in the center of the table and the rest of the deck is stacked face-down next to it. Each player builds sets of three or four matching cards from his/her hand. Matching sets are cards with equivalent fractions.

- The play moves in a clockwise direction starting with the player on the dealer's left. Each player's turn starts by drawing a card, either the top card of the deck or the top card of the discard pile. Then, if the player has any sets, s/he may (but is not required to) lay them down for everyone to see. If there is one card that matches a set that someone else has played, the player may also lay it down during their turn. Finally, the player must discard one card face-up on the top of the discard pile.

- If all of the cards in the deck are used before a player goes out, the discard pile—except for the top card—can be shuffled and used as the deck.

- Play ends when a player discards his/her last card. At this time, each player scores one point for each card they have laid down and loses one point for each card they still hold. The player who goes out first earns seven extra points.

- Play continues until one player earns 50 points.

4. Challenge: Create another game that can be played with your cards. Write the rules and play with your classmates.

ORDER IT!

Play this game with a partner.

Need:

- 2 or more players
- 32 or more Fraction Cards (Use cards created for Fraction Rummy, R2, or cut up R3)

The object of this game is to get five numbers in a row, in order, from least value to greatest value.

Once a card is placed on the table face up, it may not be moved to another location. However, a new card may be placed on top of it.

- Shuffle all the cards and place the cards face down in a pile.
- To begin, put 5 cards face-up in the center, in the order they are drawn.
- The first player draws a card from the pile and places it **on top of** one of the existing face-up cards. If all of the cards are now in order from least to greatest, then the player wins. If not, then play continues.
- The next player draws a card from the pile and places it **on top of** one of the existing face-up cards. If all the cards are now in order from least to greatest, then the player wins. If not, then play continues until all five cards are in order from least to greatest.

In order to win, a player must convince his or her opponents with a reasonable argument that the cards are in order, with each card less than or equal to the card that follows it.

1. Play two rounds of Order It! Record one of the winning ordered card sequences here.

   __________ __________ __________ __________ __________

2. Explain how you know the numbers are in order.

POSTER PROBLEM: ORDERING FRACTIONS

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is _______.
- Each group will have a different colored marker. Our group marker is _________.

Part 2: Do the problems on the posters by following your teacher's directions.

	A		B		C		D	
Poster 1 (or 5)	0	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{6}{8}$	$\frac{6}{10}$	$\frac{1}{3}$	$\frac{7}{8}$
Poster 2 (or 6)	0	1	$\frac{3}{4}$	$\frac{3}{7}$	$\frac{3}{9}$	$\frac{1}{6}$	$\frac{19}{20}$	$\frac{5}{10}$
Poster 3 (or 7)	0	1	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{3}{5}$	$\frac{13}{14}$	$\frac{2}{5}$	$\frac{3}{4}$
Poster 4 (or 8)	0	1	$\frac{3}{4}$	$\frac{3}{6}$	$\frac{3}{10}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{2}{3}$

A. Copy the eight numbers on your poster. Make a number line that is nearly the width of your paper, and put the numbers 0 and 1 on it. 0 at the far left and 1 at the far right.

B. Place the FIRST TWO FRACTIONS ONLY on the number line. Explain in writing how you decided their relative placement.

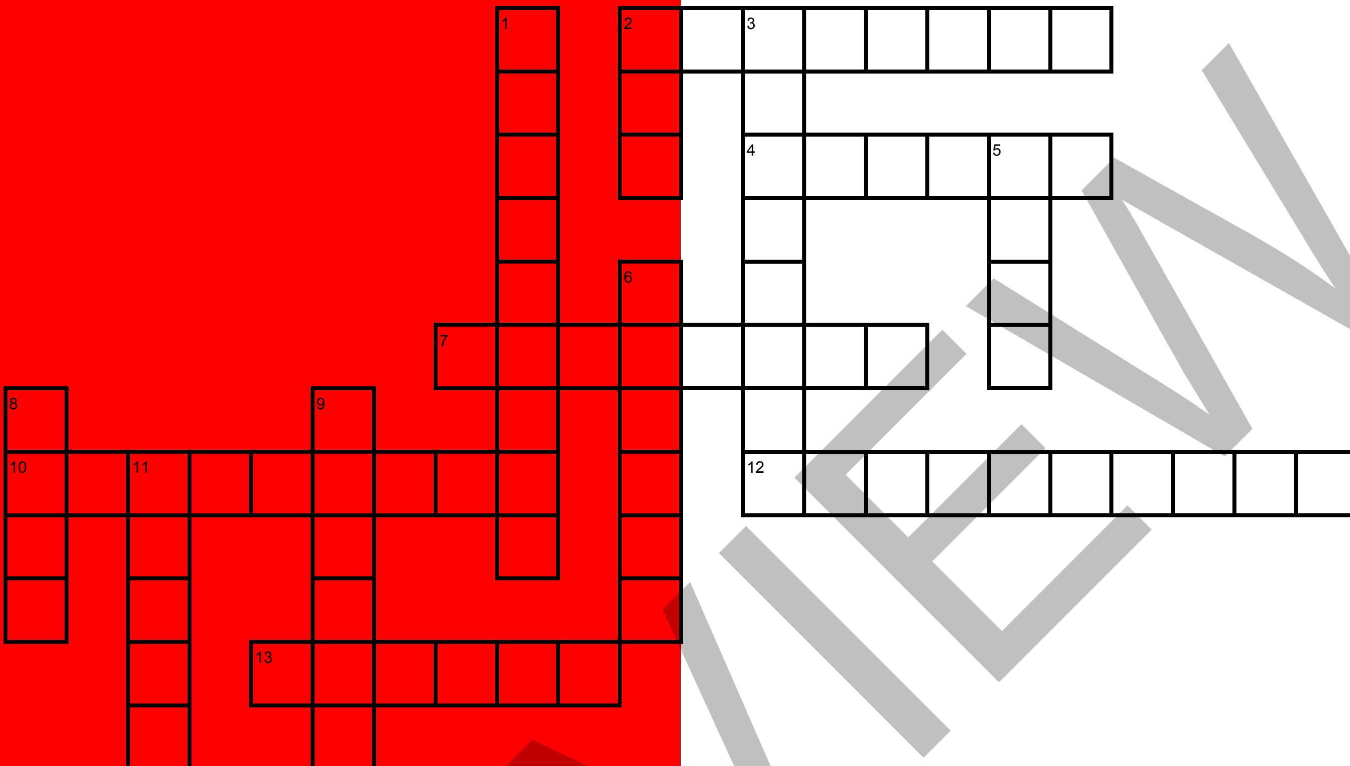
C. Place the NEXT TWO FRACTIONS ONLY on the number line. Explain in writing how you decided their relative placement.

D. Place the LAST TWO FRACTIONS ONLY on the number line. Explain in writing how you decided their relative placement.

Part 3: Return to your start poster.

- Check all the work on the poster.
- Be prepared to share one strategy that was explained particularly well.
- Rewrite one strategy that could be stated better here.

VOCABULARY REVIEW



Across

2 A fraction is in ___ form when the numerator and denominator have no factors in common (other than 1).

4 A(n) ___ model for fractions uses a number line or fraction strip.

7 A(n) ___ fraction is greater than or equal to 1.

10 A(n) ___ is the top number of a fraction.

12 ___ fractions look different but have the same value.

13 $\frac{3}{4}$ and $\frac{1}{4}$ have a ___ denominator.

Down

1 A(n) ___ is an easily recognized fraction, like $\frac{1}{2}$.

2 A(n) ___ model for fractions uses separate objects.

3 20 is a ___ of 4 because $4 \times 5 = 20$.

5 A(n) ___ model for fractions shows the whole figure cut into equal sized parts.

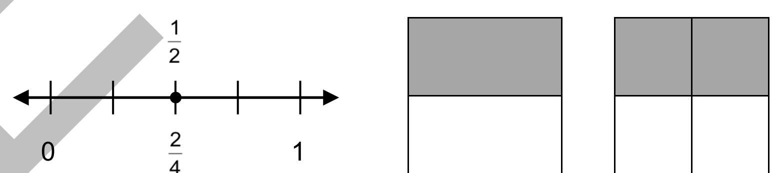
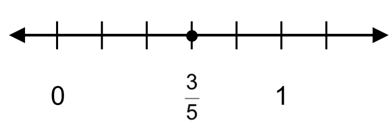
6 A(n) ___ fraction is between 0 and 1.

8 A(n) ___ fraction has a numerator of 1.

9 A(n) ___ is a divisor of a number.

11 A combination of a whole number and a fraction is a(n) ___ number.

DEFINITIONS, EXPLANATIONS, AND EXAMPLES

Word or Phrase	Definition
benchmark fraction	A benchmark fraction refers to a fraction that is easily recognizable. It is easily identified on the number line, and it is more commonly used in everyday experiences. Some benchmark fractions might be $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{2}{3}, \frac{3}{4}$.
common denominator	A common denominator of two or more fractions is a number that is divisible by each of the denominators of the fractions. A common denominator of the fractions $\frac{1}{6}$ and $\frac{3}{4}$ is 24, since 24 is divisible by both 6 and 4. Another common denominator of these fractions is 36. The least common denominator of these fractions is 12.
denominator	The denominator of the fraction $\frac{a}{b}$ is b. The denominator of $\frac{3}{7}$ is 7.
equivalent fractions	The fractions $\frac{a}{b}$ and $\frac{c}{d}$ are equivalent if they represent the same point on the number line. This occurs if the results of the division problems $a \div b$ and $c \div d$ are equal. Since $\frac{1}{2} = 1 \div 2 = 0.5$ and $\frac{2}{4} = 2 \div 4 = 0.5$, the fractions $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent. Pictorially: 
factor of a number	A factor of a number is a divisor of the number. The factors of 12 are 1, 2, 3, 4, 6, and 12.
fraction	A fraction is a number expressible in the form $\frac{a}{b}$ where a is a whole number and b is a positive whole number. The fraction $\frac{3}{5}$ is represented by the dot on the number line. 

Word or Phrase	Definition
improper fraction	An <u>improper fraction</u> is a fraction of the form $\frac{m}{n}$, where $m \geq n$ and $n > 0$. The fractions $\frac{3}{2}$, $\frac{17}{4}$, $\frac{9}{9}$ and $\frac{32}{16}$ are improper fractions.
mixed number	A <u>mixed number</u> is an expression of the form $n\frac{p}{q}$, which is a shorthand for $n + \frac{p}{q}$, where n, p, and q are positive whole numbers. The mixed number $4\frac{1}{4}$ ("four and one fourth") is shorthand for $4 + \frac{1}{4}$. It should not be confused with the product $4 \bullet \frac{1}{4} = 1$.
Multiple of a number	A <u>multiple of a number</u> m is a number of the form $k \bullet m$ for any integer k. The numbers 5, 10, 15, and 20 are multiples of 5, since $1 \bullet 5 = 5$, $2 \bullet 5 = 10$, $3 \bullet 5 = 15$, and $4 \bullet 5 = 20$.
multiplication property of 1	The <u>multiplication property of 1</u> states that $a \bullet 1 = 1 \bullet a = a$ for all numbers a. In other words, 1 is a <u>multiplicative identity</u>. The multiplication property of 1 is sometimes called the <u>multiplicative identity property</u>. $4 \bullet 1 = 4$, $1 \bullet 25 = 25$, $\frac{1}{2} \bullet \boxed{\frac{4}{4}} = \frac{4}{8}$ In the third equation above, since we are multiplying by 1 in the form of $\frac{4}{4}$, we refer to it as the "big 1."
natural number	The <u>natural numbers</u> are the numbers 1, 2, 3, 4, ….
numerator	The <u>numerator</u> of the fraction $\frac{a}{b}$ is a. The numerator of $\frac{3}{7}$ is 3.
proper fraction	A <u>proper fraction</u> is a fraction of the form $\frac{m}{n}$, where $1 \leq m < n$. The fractions $\frac{1}{2}$ and $\frac{5}{6}$ are proper fractions.
unit fraction	A <u>unit fraction</u> is a fraction of the form $\frac{1}{m}$, where m is a natural number. The unit fractions are $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$.
whole number	The <u>whole numbers</u> are the numbers 0, 1, 2, 3, 4, ….

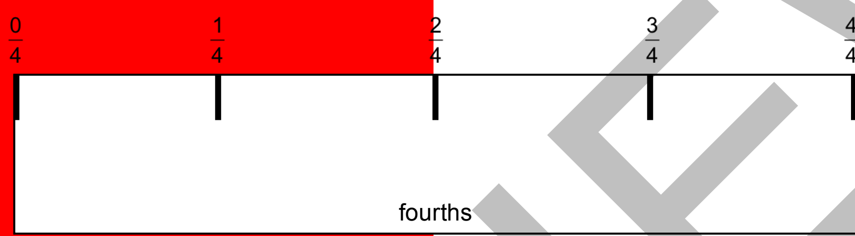
Models for Fractions

Linear Model

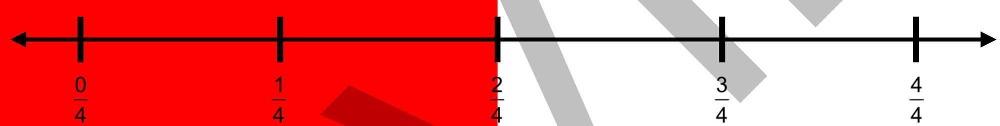
One useful model for fractions is a linear model. In a linear model, the whole (or unit) is represented by a specified interval on a number line. Then fractions are represented as lengths of intervals in comparison to the length of the whole.

The paper strip pictured below represents 1 whole unit of length, divided into fourths (four equal units of length). Notice that the very left edge represents zero (zero-fourths), and the very right edge represents 1 (four-fourths). Rulers work in much the same way.

This strip is marked off in fourths.



This edge of the strip represents a linear model.

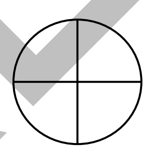


One common error in working with linear models is to start counting "1" at the very left edge, or to count tick marks instead of "spaces." Notice that it requires 5 tick marks to make 4 spaces.

Area Model

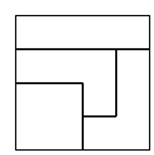
Another useful model for fractions is an area model. In an area model, the whole is represented as the area of some specified shape. Then fractions are represented as areas of shapes that can be compared to the whole.

If the circle is defined as 1 whole, and each part is of equal area, then each part represents $\frac{1}{4}$ of the whole.



These parts happen to be congruent as well.

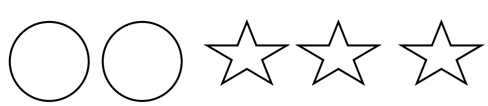
If the rectangle is defined as 1 whole, and each part is of equal area, then each part represents $\frac{1}{4}$ of the whole.



These parts are not all congruent, but they still have equal area.

Set Model

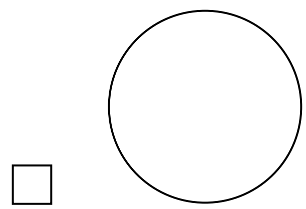
A third useful model for fractions is a set model. Set models are based on numbers of objects in a set, not their area. For example, in this diagram, $\frac{2}{5}$ of the objects are circles, and $\frac{3}{5}$ of the objects are stars.



Sense-Making Strategies for Comparing and Ordering Fractions

Examples	Name	Ordering Strategy
$\frac{1}{3} < \frac{1}{2} < \frac{3}{4}$	Benchmark fractions	Benchmark fractions are fractions that are easily recognizable, such as $\frac{1}{2}$. For example, $\frac{3}{8} < \frac{1}{2}$, because 3 is less than half of 8.
$\frac{1}{8} < \frac{1}{5} < \frac{1}{4}$	Unit fractions	When comparing unit fractions, the fraction with the greater denominator has a lesser value. Think: "When you are very hungry, do you want to share a pizza equally among 8 friends or 4 friends? In which situation do you get more pizza?"
$\frac{3}{8} < \frac{3}{5} < \frac{3}{4}$	Fractions with common numerators	When comparing fractions with common numerators, the fraction with the greater denominator has a lesser value. Using similar reasoning as above: "If ONE-fourth is greater than ONE-eighth, then THREE-fourths must be greater than THREE-eighths."
$\frac{1}{12} < \frac{3}{12} < \frac{8}{12}$	Fractions with common denominators	When comparing fractions with common denominators, the fraction with the greater numerator has a greater value. Think: "A pizza is divided into 8 equal parts. If you eat 1 slice and your friend eats 3 slices, who ate more pizza?"

In the comparisons above, we assume that all fractions in each example refer to the same whole. This is important because $\frac{1}{2}$ of the circle to the right has a greater area than $\frac{9}{10}$ of the square to the right.



The "Big One"

The "big 1" is a notation for 1 (multiplicative identity) in the form of a fraction $\frac{n}{n}$ ($n \neq 0$).

$$1 = \frac{1}{1} = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5} = \ldots$$

We can use the following picture to help remind us that these fractions are equivalent to 1:

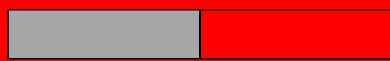
$$1 = \frac{8}{8}$$

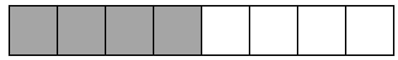
The "big 1" can be used to show equivalence of fractions. For example,

$$\frac{2}{5} \times \frac{10}{10} = \frac{20}{50} \quad \text{or} \quad \frac{20}{50} \div \frac{10}{10} = \frac{2}{5}.$$

Diagrams that Show Equivalent Fractions

These diagrams illustrate that $\frac{1}{2}=\frac{4}{8}$. In the second diagram, each half is split into four parts, but the size of the whole does not change, nor does the amount shaded.

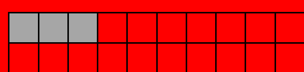


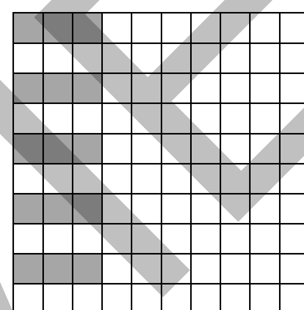


Using the "big 1," this equivalence can be written:

$$\frac{1}{2}\bullet\boxed{\frac{4}{4}}=\frac{4}{8}.$$

These diagrams illustrate that $\frac{3}{20}=\frac{15}{100}$. In the second diagram, the pattern is repeated five times. The fractional part remains the same as the size of the whole changes.

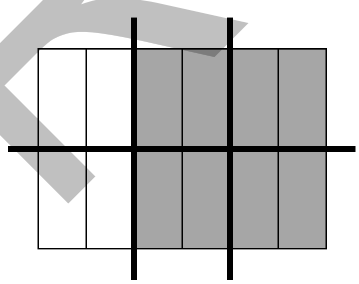


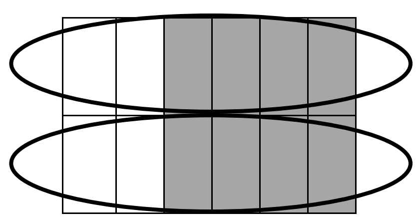


Using the "big 1," this equivalence can be written:

$$\frac{3}{20}\bullet\boxed{\frac{5}{5}}=\frac{15}{100}.$$

Each of these two diagrams illustrate that $\frac{8}{12}=\frac{4}{6}$. The first diagram shows groups of 2 created by heavy lines. The second diagram shows two equal groups using circles.





Using the "big 1," this equivalence can be written:

$$\frac{8}{12}\div\boxed{\frac{2}{2}}=\frac{4}{6}.$$

Improper Fractions and Mixed Numbers

Improper fractions may be represented as mixed numbers and vice versa.

Example: Change $5\frac{2}{3}$ into an improper fraction.

Since $5\frac{2}{3} = 5 + \frac{2}{3}$ and $5 = \frac{15}{3}$, $5\frac{2}{3} = \frac{15}{3} + \frac{2}{3} = \frac{17}{3}$.

Here is a shortcut.

Think: 5 times 3 (denominator) is 15, and 15 + 2 (numerator) = 17. So $5\frac{2}{3} = \frac{17}{3}$.

5 x 3 = 15 (15 thirds)	$5\frac{2}{3}$	15 + 2 = 17 (17 thirds)

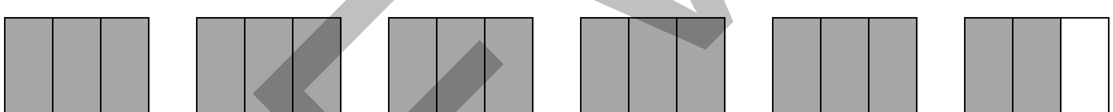
Example: Change $\frac{17}{3}$ into a mixed number.

Recall that $\frac{17}{3}$ can be written as 17 ÷ 3

17 ÷ 3 = 5, with a remainder of 2

$\frac{17}{3} = \frac{15}{3} + \frac{2}{3} = 5 + \frac{2}{3} = 5\frac{2}{3}$

$\frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{2}{3} = \frac{17}{3}$



$1 + 1 + 1 + 1 + 1 + \frac{2}{3} = 5\frac{2}{3}$

Fractions in "Simplest Form"

To write a fraction in its simplest form, divide the numerator and denominator by a common factor. For this you may use the "big 1." Continue the process until the numerator and denominator have no factors (except 1) in common.

- To simplify $\frac{12}{30}$, first find a factor that 12 and 30 have in common. Since 2 is a factor of 12 and 30, divide the numerator and denominator by 2 in the form of the "big 1."

$$\frac{12}{30} = \frac{12}{30} \div \frac{2}{2} = \frac{6}{15}.$$

The result is $\frac{6}{15}$. Since 3 is a factor of 6 and 15, divide by "the big 1" again.

$$\frac{6}{15} \div \frac{3}{3} = \frac{2}{5}$$

Since 2 and 5 have no factors in common (except 1), $\frac{2}{5}$ is the simplest form of $\frac{12}{30}$.

- Another way to simplify $\frac{12}{30}$ is to divide numerator and denominator by 6 in the form of the "big 1.

$$\frac{12}{30} \div \frac{6}{6} = \frac{2}{5}$$

This simplification is performed in one step because 6 is the greatest factor that 12 and 30 have in common. In other words, 6 is the "greatest common factor" (GCF) of 12 and 30.

Renaming Fractions with Common Denominators

To find a common denominator for two fractions, $\frac{3}{4}$ and $\frac{5}{6}$:

Method 1:
A common denominator can always be found by multiplying the two original denominators (4 • 6 = 24). Since 24 is a multiple that the denominators share in common, multiply the numerator and denominator of each fraction by an appropriate missing factor in the form of the "big 1" so that the resulting denominator is 24.

$$\frac{3}{4} \times \frac{6}{6} = \frac{18}{24} \qquad \frac{5}{6} \times \frac{4}{4} = \frac{20}{24}$$

Method 2:

- Make a list of multiples for each denominator.
- Choose a multiple that the denominators share as the common denominator.
- Multiply each fraction by an appropriate missing factor in the form of the "big 1" so that the result is the common denominator.

Multiples of 4: 4, 8, $\boxed{12,}$ 16, 20, 24…

Multiples of 6: 6, $\boxed{12,}$ 18, 24, 30, 36…

Since 12 is a multiple that the denominators share in common, multiply the numerator and denominator of each fraction by an appropriate missing factor in the form of the "big 1" so that the resulting denominator is 12.

$$\frac{3}{4} \times \frac{3}{3} = \frac{9}{12} \qquad \frac{5}{6} \times \frac{2}{2} = \frac{10}{12}$$

This computation results in smaller numbers in the fractions because 12 is the least mutiple that 4 and 6 share in common. In other words, 12 is the "least common multiple" (LCM) of 4 and 6.

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